Computer-aided cryptographic proofs

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Computer-aided cryptography

Develop tool-assisted methodologies for helping the design, analysis, and implementation of cryptographic constructions (primitives and protocols)

Building on formal methods

- program analysis and verification/program synthesis
- compilation (certifying compilation/verified compilation)
- logic
- etc

Potential benefits

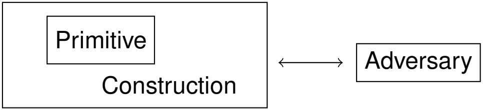
Formal methods for cryptography

- higher assurance
- smaller gap between provable security and crypto engineering
- new proof techniques

Cryptography for formal methods

- Challenging and non-standard examples
- New theories and applications

Provable security



For every adversary $\mathcal{A}$ executing in time $t_{\mathcal{A}}$ there exists an adversary executing in time $t_{\mathcal{B}}$ such that

$$Pr[\mathcal{A} \text{ breaks } C] \leq Pr[\mathcal{B} \text{ breaks P}] + \epsilon$$

and

$$t_{\mathcal{A}} \leq t_{\mathcal{B}} + M$$

EasyCrypt[1]

Domain-specific proof assistant

- proof goals tailored to reductionist proofs
- proof tools support common proof techniques (bridging steps, failure events, hybrid arguments, eager sampling. . .)

Control and automation from state-of-art verification

- interactive proof engine and mathematical libraries (a la Coq/ssreflect)
- back-end to SMT solvers

Many case studies:

- Encryption, signatures, key exchange, zero-knowledge, multi-party and verifiable computation, SHA3, voting, KMS

[1] Gilles Barthe, Benjamin Grégoire, Sylvain Heraud, Santiago Zanella Béguelin: Computer-Aided Security Proofs for the Working Cryptographer. CRYPTO 2011

Probabilistic Relational Hoare logic[2]

- Code-based approach

$$
\begin{array}{rcll}
\mathcal{C} & ::= & \text{Skip} & \text{skip} \\
& | & \mathcal{V} \leftarrow \mathcal{E} & \text{assignment} \\
& | & \mathcal{V} \overset{\$}{\leftarrow} \mathcal{D} & \text{random sampling} \\
& | & \mathcal{C}; \mathcal{C} & \text{sequence} \\
& | & \text{if } \mathcal{E} \text{ then } \mathcal{C} \text{ else } \mathcal{C} & \text{conditional} \\
& | & \text{while } \mathcal{E} \text{ do } \mathcal{C} & \text{while loop} \\
& | & \mathcal{V} \leftarrow \mathcal{F}(\mathcal{E}, \ldots, \mathcal{E}) & \text{procedure (oracle/adv) call}
\end{array}
$$

- Game-playing technique: $\models \{\phi\}\ c_1 \sim c_2\ \{\psi\}$ where ϕ and ψ are relations on states
- Validity:

$$(m_1, m_2) \in \phi \Longrightarrow \blacktriangleleft_{\psi} \langle [\![c_1]\!]_{m_1} \,\&\, [\![c_2]\!]_{m_2} \rangle$$

[2]Gilles Barthe, Benjamin Grégoire, Santiago Zanella Béguelin: Formal certification of code-based cryptographic proofs. POPL 2009

Probabilistic couplings

Let $\mu_1, \mu_2 \in \text{Dist}(A)$ and $R \subseteq A \times A$. Let $\mu \in \text{Dist}(A \times A)$.

- μ is a coupling for (μ_1, μ_2) iff $\pi_1(\mu) = \mu_1$ and $\pi_2(\mu) = \mu_2$
- μ is a R-coupling for (μ_1, μ_2) if moreover $\Pr_{y \leftarrow \mu}[y \notin R] = 0$

Let μ be a R-coupling for (μ_1, μ_2)

- Bridging step: if R is equality, then for every event X,

$$\Pr_{z \leftarrow \mu_1}[X] = \Pr_{z \leftarrow \mu_2}[X]$$

- Failure Event: If $x \; R \; y$ iff $\neg F(x) \Rightarrow x = y$ and $F(x) \Leftrightarrow F(y)$, then for every event X,

$$|\Pr_{z \leftarrow \mu_1}[X] - \Pr_{z \leftarrow \mu_2}[X]| \leq \max\left(\Pr_{z \leftarrow \mu_1}[F], \Pr_{z \leftarrow \mu_2}[F]\right)$$

- Reduction: If $x \; R \; y$ iff $F(x) \Rightarrow G(y)$, then

$$\Pr_{x \leftarrow \mu_2}[G] \leq \Pr_{y \leftarrow \mu_1}[F]$$

A program logic for probabilistic couplings

Rules for sequence and conditionals:

$$\frac{\models \{\psi\}\ c_1 \ \sim\ c_2\ \{\Theta\} \quad \models \{\Theta\}\ c_1' \ \sim\ c_2'\ \{\phi\}}{\models \{\psi\}\ c_1 ; c_1' \ \sim\ c_2 ; c_2'\ \{\phi\}}$$

$$\frac{\models \{\psi \wedge b_1\}\ c_1 \ \sim\ c_2\ \{\phi\} \quad \models \{\psi \wedge \neg b_1\}\ c_1' \ \sim\ c_2'\ \{\phi\} \quad \psi \implies b_1 = b_2}{\models \{\psi\}\ \text{if } b_1 \text{ then } c_1 \text{ else } c_1' \ \sim\ \text{if } b_2 \text{ then } c_2 \text{ else } c_2'\ \{\phi\}}$$

Rules for random sampling

$$\frac{\blacktriangleleft_{\varphi} \langle [\![\mu_1]\!] \ \&\ [\![\mu_2]\!] \rangle \qquad \varphi \triangleq \forall v_1 : T_1,\ v_2 : T_2,\ \varphi \implies \psi[v_1/x_1][v_2/x_2]}{\models \{\varphi\}\ x_1 \xleftarrow{\$} \mu_1 \ \sim\ x_2 \xleftarrow{\$} \mu_2\ \{\psi\}}$$

$$\frac{}{\models \{\forall v_1 \in \mathrm{supp}(d_1), \psi[v_1/x_1]\}\ x_1 \xleftarrow{\$} d_1 \ \sim\ \mathrm{skip}\ \{\psi\}}$$

Rules for adversary (simplified)

$$\frac{}{\models \{e_1 = e_2 \wedge \varphi\}\ x_1 \leftarrow \mathcal{A}(e_1) \ \sim\ x_2 \leftarrow \mathcal{A}(e_2)\ \{x_1 = x_2 \wedge \varphi\}}$$

From algorithms to implementations

- We need cryptographic libraries that we can trust
- Correct implementation frequently remain vulnerable to attacks, inc. implementation bug attacks and side channel attacks
- Efficiency considerations often force developers to carry out very aggressive optimizations

Can we carry guarantees to implementations?

Question: which guarantees?

- Fast
- Correct (functional correctness)
- Side-channel resistant (constant-time)
- Provably secure

Question: which implementations

Source

- Portable
- Convenient software-engineering abstractions
- Readable, maintainable

Assembly

- Efficient
- Controlled (instruction selection and scheduling)
- Predictable

A gap between source and assembly languages

- **Assembly** is not programmer/verifier friendly
  - Harder to understand
  - More error prone
  - Harder to prove / analyze
- **Source** is not security/efficiency friendly
  - Trust compiler
  - Certified compilers are less efficient
  - Optimizing compilers can break side-channel resistance

Jasmin[3],[4] [5]

Fast and formally verified assembly code

- Source language: assembly in the head with formal semantics
  $\Longrightarrow$ programmer & verification friendly
- Compiler: predictable & formally verified (in Coq)
  $\Longrightarrow$ programmer has control and no compiler security bug
- Verification toolchain (based on EasyCrypt):
  - security
  - safety
  - side channel resistance (constant-time)
  - functional correctness

Case studies

ChaCha20, Poly1305, Curve25519, SHA3

[3] José Bacelar Almeida, Manuel Barbosa, Gilles Barthe, Arthur Blot, Benjamin Grégoire, Vincent Laporte, Tiago Oliveira, Hugo Pacheco, Benedikt Schmidt, Pierre-Yves Strub: Jasmin: High-Assurance and High-Speed Cryptography. CCS, 2017

[4] José Bacelar Almeida, Manuel Barbosa, Gilles Barthe, Benjamin Grégoire, Adrien Koutsos, Vincent Laporte, Tiago Oliveira, Pierre-Yves Strub: The Last Mile: High-Assurance and High-Speed Cryptographic Implementations. S&P, 2020

[5]

Jasmin language

Zero-cost abstractions:

- Variable names
- Arrays
- Conditional and loops
- Inline functions

Control:

- *For* loops are unrolled. *While* loops are not unrolled
- Spilling and instruction scheduling is controlled manually
- Instruction-set architectures are available (often in user-friendly form)

Initialization of Chacha20 state

```
inline fn init(reg u64 key nonce, reg u32 counter) → stack u32[16]
{
  inline int i;
  stack u32[16] st;
  reg u32[8] k;
  reg u32[3] n;

  st[0] = 0x61707865;
  st[1] = 0x3320646e;
  st[2] = 0x79622d32;
  st[3] = 0x6b206574;

  for i=0 to 8 {
    k[i] = (u32)[key + 4∗i];
    st[4+i] = k[i];
  }

  st[12] = counter;

  for i=0 to 3 {
    n[i] = (u32)[nonce + 4∗i];
    st[13+i] = n[i];
  }

  return st;
}
```

Compiler: efficiency, predictability and verification-preserving

Preservation of functional correctness (proved in Coq)

$$\forall\, p\, p' \;\; \mathrm{compile}(p) = \mathrm{ok}(p') \Rightarrow$$
$$\forall\, f \;\in \mathrm{exports}(p) \Rightarrow$$
$$\forall\, m\; \text{enough-stack-space}(f, p', m) \Rightarrow$$
$$\forall\, v_a\; v_r\; m'.\; p : f, v_a, m \Downarrow v_r, m' \Rightarrow p' : f, v_a, m \Downarrow v_r, m'$$

Preservation of side-channel resistance (proved on paper)

$$\forall\, p\, p'.\; \mathrm{compile}(p) = \mathrm{ok}(p') \Rightarrow$$
$$\forall\, f \;\in \mathrm{exports}(p) \Rightarrow$$
$$\forall\, m_1\;\; m_2 \;\; \wedge_{i=1,2}\, \text{enough-stack-space}(f, p', m_i) \wedge m_1 \sim m_2 \Rightarrow$$
$$\forall\, v_a.\; p : f, v_a, m_1 \rightsquigarrow \ell_1 \wedge p : f, v_a, m_2 \rightsquigarrow \ell_2 \Rightarrow \ell_1 = \ell_2 \Rightarrow$$
$$\forall\, v_a.\; p' : f, v_a, m_1 \rightsquigarrow \ell_1 \wedge p' : f, v_a, m_2 \rightsquigarrow \ell_2 \Rightarrow \ell_1 = \ell_2$$

Constant-time programming

Software-based countermeasure against cache-based timing attacks:

- control-flow should not depend on secret data
- memory accesses should not depend on secret data

Rationale: crypto implementations without this property are vulnerable

Secure compilation

- Can we reason about constant-time at the source level?
- Do compilers preserve constant-time?

Preservation of constant-time

Counter-examples

Before

```
int cmove(int x, int y, bool b) {
  return x + (y—x) * b;
}
```

After

```
int cmove(int x, int y, bool b) {
  if (b) {
    return y;
  } else {
    return x;
  }
}
```

Before

```
long long llmul(long long x, long long y) {
  return x * y;
}
```

After

```
long long llmul(long long x, long long y) {
  long a = High(x);
  long c = High(y);
  if (a | c) {
    . . .
  } else {
    return Low(x) * Low(y);
  }
}
```

However, many compiler optimizations do preserve “constant-time”:

- (Adjusted) CompCert preserves constant-time (Coq proof)
- Jasmin preserves constant-time (paper proof)

Constant-time implies system-level security[6]

[6]Gilles Barthe, Gustavo Betarte, Juan Diego Campo, Carlos Daniel Luna, David Pichardie: System-level Non-interference for Constant-time Cryptography. CCS 2014

Proving functional correctness

- Using Hoare Logic: $\{P\}\ c\ \{Q\}$
  Interpretation:
  $$P\ m \Rightarrow m \Downarrow^{c} m' \Rightarrow Q\ m'$$
- Using Relational Hoare Logic: $\{P\}\ c_1 \sim c_2\ \{Q\}$
  Interpretation:
  $$P\ m_1\ m_2 \Rightarrow m_1 \Downarrow^{c_1} m_1' \Rightarrow m_2 \Downarrow^{c_2} m_2' \Rightarrow Q\ m_1'\ m_2'$$

Example:

- c_1 is the reference implementation (the specification)
- c_2 is the optimized implementation
  $$\{\text{args}\langle 1\rangle = \text{args}\langle 2\rangle\}\ c_1 \sim c_2\ \{\text{res}\langle 1\rangle = \text{res}\langle 2\rangle\}$$

EasyCrypt already provides Hoare Logic and Relational Hoare Logic

Functional correctness by game hopping

We have built an EasyCrypt model for Jasmin

Jasmin compiler is able to translate programs into the EasyCrypt syntax

We perform functional correctness proofs by game hopping:

$$c_{\mathrm{ref}} \sim c_1 \sim \ldots \sim c_{\mathrm{opt}}$$

Game hopping: example Chacha20

Chacha20 is a stream cipher that iterate a *body* on all block of the message

Reference

```
while (i < len) {
  chacha_body;
  i += 1;
}
```

Loop tiling

```
while (i < len + 4) {
  chacha_body;
  chacha_body;
  chacha_body;
  chacha_body;
  i += 4;
}
chacha_end
```

Scheduling

```
while (i < len + 4) {
  chacha_body4_swapped;
  i += 4;
}
chacha_end
```

Vectorization

```
while (i < len + 4) {
  chacha_body4_vectorized;
  i += 4;
}
chacha_end
```

Side channel resistance

Verify using Relational Hoare Logic on instrumented code:

$$\text{if } t \text{ then } c_1 \text{ else } c_2 \qquad\qquad \text{leaks} \leftarrow t :: \text{leaks}; \text{if } t \text{ then } c_1 \text{ else } c_2$$

c is constant time if its model $\overline{c}$ verifies the relational property:

$$\models \{=_{\textit{public inputs}}\}\ c \ \sim\ c\ \{\text{leaks}\langle 1\rangle = \text{leaks}\langle 2\rangle\}$$

Inspired and equivalent to method implemented in ctverif[7]

[7]Jose Bacelar Almeida, Manuel Barbosa, Gilles Barthe, François Dupressoir, Michael Emmi: Verifying Constant-Time Implementations. USENIX Security Symposium 2016

Verified implementations

- FOR EVERY adversary that breaks implementation,
- IF implementation is safe and constant-time,
- AND implementation is correct with respect to algorithm,
- THERE EXISTS an adversary that breaks the algorithm

Moreover implementation is fast

Other approaches

- HACL*: Verified functional correctness and constant-time of source code. Broad coverage. Portable, but compiler trade-off
- Vale: Verified functional correctness and constant-time of assembly code. Broad coverage.
- Cryptoline: Verified functional correctness of assembly code. Broad coverage.
- FiatCrypto: Verified functional correctness of assembly code. Focus on arithmetic routines.

Future work

- More architectures
- Machine-checked proof of preservation of constant time
- Automation of equivalence proofs
- More examples

Summary

Foundations and tools for high-assurance cryptography

- Provable security
- Practical cryptography
- Reducing the gap between security proofs and implementations

Broader vision

- Scalable program verification for mainstream languages
- Efficient verified compilers for mainstream languages
- How to protect cryptography against micro-architectural attacks?