A Reaction Attack against Cryptosystems based on LRPC Codes

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Outline

Post-quantum cryptography

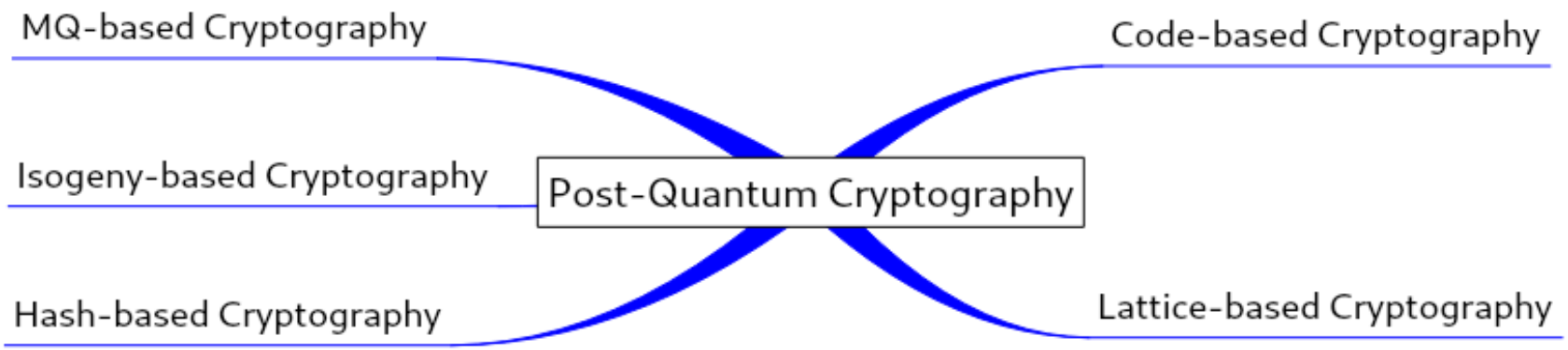
Why do we need post-quantum cryptography?

Shor's Algorithm solves in polynomial time:

- Integer factorization; RSA is dead.
- The discrete-logarithm problem in finite fields; DSA is dead.
- The discrete-logarithm problem on elliptic curves; ECDSA is dead.

Post-quantum cryptography

What is post-quantum cryptography?



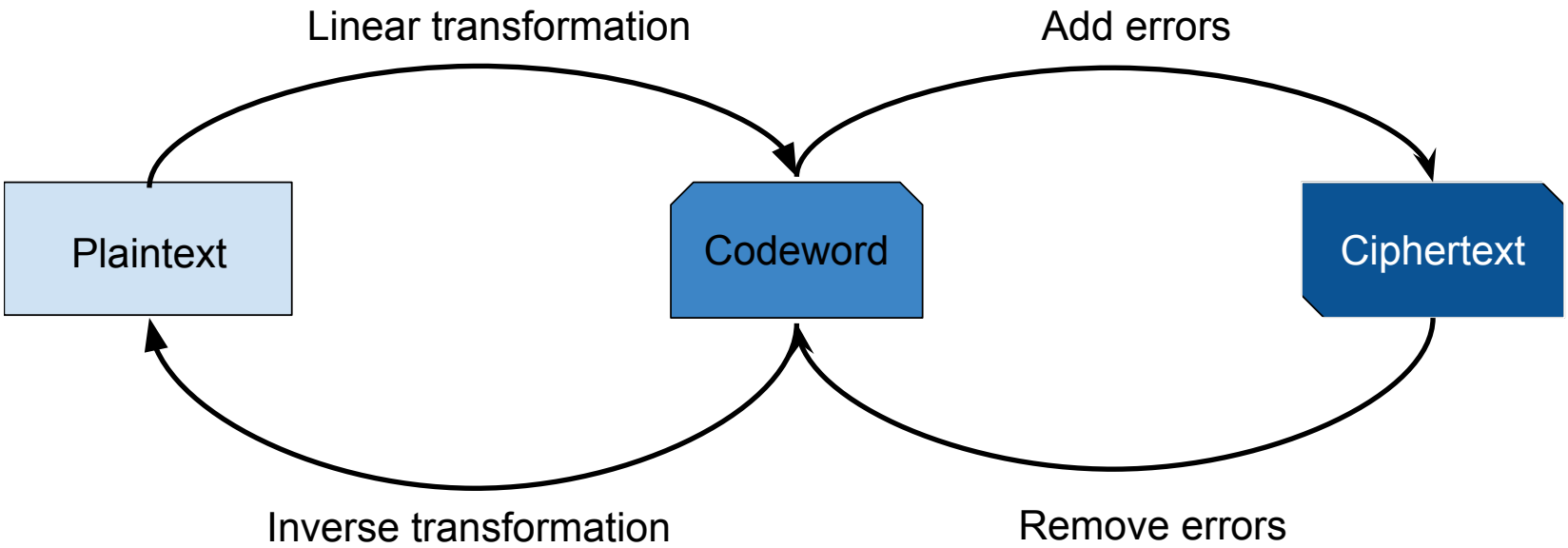
Post-quantum cryptography

Timeline

- 2016: NIST calls for submissions to "Post-Quantum Cryptography Standardization Project".
- 2017: NIST receives 69 proper submissions.
- 2018-19: NIST 2nd round of proposals with 26 proposals.
- 17 code-based in the 1st round; 7 code-based in 2nd round.

Code-based Cryptography

Code-based cryptography in a nutshell



- Originally proposed by McEliece in 1978;
- It uses a linear code:
  - Goppa codes;
  - LDPC/MDPC;
  - Rank Metric (LRPC);
  - Several others.

Rank Metric Codes

A *Low-Rank Parity-Check (LRPC) code*

A LRPC $\mathcal{C}$ over $\mathbb{F}_{q^m}$ of length n, dimension k and rank d is described by an $(n-k) \times n$ parity-check matrix

$$\mathbf{H} = \{h_{i,j}\} \in \mathbb{F}_{q^m}^{(n-k)\times n},$$

- Each coefficient $h_{i,j}$ can be written as

$$h_{i,j} = \sum_{l=1}^{d} h_{i,j,l} F_l, \quad h_{i,j,l} \in \mathbb{F}_q,$$

each $F_i \in \mathbb{F}_{q^m}$, and $F = \langle F_1, F_2, \cdots, F_d \rangle$ is a $\mathbb{F}_q$ subspace of $\mathbb{F}_{q^m}$.

Decoding LRPC codes

How to decode LRPC codes?

Let $\mathbf{s} = (s_1, \ldots, s_{n-k}) \in \mathbb{F}_{q^m}^{n-k}$ be the *syndrome* of $\mathbf{e}$, i.e. $\mathbf{He}^\top = \mathbf{s}$.

Decoding: Recover $\mathbf{e}$ from the knowledge of $\mathbf{s}$.

Crucial facts:

- If $h_{i,j} \in F = \langle F_1, F_2, \cdots, F_d \rangle$ and $e \in E = \langle E_1, E_2, \cdots, E_r \rangle$ then

$$s_i \in \langle F_1 E_1, F_1 E_2, \ldots, F_d E_r \rangle$$

- Assume $S = \langle s_1, s_2, \ldots, s_{n-k} \rangle = \langle F_1 E_1, F_1 E_2, \ldots, F_d E_r \rangle$ then:

1. Set $S_i = F_i^{-1}.S$. Then

$$S_i = F_i^{-1}.\langle \ldots F_i E_1, F_i E_2, \ldots, F_i E_r \ldots \rangle \Rightarrow E = \langle E_1, E_2, \cdots, E_r \rangle \subset S_i$$

2. Find $E = S_1 \cap S_2 \cap \cdots \cap S_d$
3. Find $\mathbf{e}$ by solving $\mathbf{He}^\top = \mathbf{s}$

Decoding of LRPC codes

When do decoding failures happen?

1. When $\mathrm{Dim}\big(\langle EF \rangle\big) < rd$: this happens with probability $P_1 = \frac{d}{q^{m-rd}}$
2. When $E \neq \bigcap_{i=1}^{d} S_i$: when $m > rd + 8$, this happens with probability $P_2 \ll 2^{-30}$
3. **When $\mathrm{Dim}\big(S\big) < rd$ this happens with probability** $P_3 = \frac{1}{q^{n-k+1-rd}}$

- In practice usually $P_1, P_2 \ll P_3$.

LRPC cryptosystems

What is a LRPC cryptosystem?

Basically any cryptosystem that

- uses LRPC codes (low rank of $\mathbf{H}_{secret}$)
- uses $\mathbf{RH}_{secret} = \mathbf{H}$ to hide the secret $\mathbf{H}_{secret}$
- relies on the **Rank syndrome decoding problem:**

$$\text{Find } \mathbf{e} \text{ such that } \mathbf{He}^\top = \mathbf{s} \text{ and } |\mathbf{e}| \leqslant r.$$

- LRPC cryptosystem [Gaborit et al.'13]
- McNie [Kim et al.'17] (NIST 1st round candidate)
- ROLLO (Rank-Ouroboros, LAKE and LOCKER) [Aguilar Melchor et al. '17] (NIST 2nd round candidate)
- Durandal [Aragon et al.'19]

Reaction attack

Reaction attack

$$\mathbf{m}_1, \mathbf{e}_1, \mathbf{c}_1 = \mathbf{m}_1\mathbf{G} + \mathbf{e}_1$$

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$$\mathbf{m}_1, \mathbf{e}_1, \mathbf{c}_1 = \mathbf{m}_1\mathbf{G} + \mathbf{e}_1 \xrightarrow{\qquad\qquad \mathbf{c}_1 \qquad\qquad} \longrightarrow$$

Reaction attack

$$\mathbf{m}_1, \mathbf{e}_1, \mathbf{c}_1 = \mathbf{m}_1\mathbf{G} + \mathbf{e}_1 \xrightarrow{\quad \mathbf{c}_1 \quad} \checkmark \leftarrow \mathit{Decode}(\mathbf{c}_1)$$

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$$\mathbf{m}_2, \mathbf{e}_2, \mathbf{c}_2$$

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$$\xrightarrow{\ldots}$$

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$\mathbf{m}_1, \mathbf{e}_1, \mathbf{c}_1 = \mathbf{m}_1\mathbf{G} + \mathbf{e}_1 \xrightarrow{\mathbf{c}_1} \checkmark \leftarrow \mathit{Decode}(\mathbf{c}_1)$

$\mathbf{m}_2, \mathbf{e}_2, \mathbf{c}_2 \xrightarrow{\mathbf{c}_2} \checkmark \leftarrow \mathit{Decode}(\mathbf{c}_2)$

$\xrightarrow{\ldots}$

$\mathbf{m}_t, \mathbf{e}_t, \mathbf{c}_t$

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$\xrightarrow{\ldots}$

$\mathbf{m}_t, \mathbf{e}_t, \mathbf{c}_t \xrightarrow{\mathbf{c}_t} \mathrm{X} \leftarrow \mathit{Decode}(\mathbf{c}_t)$

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$\xrightarrow{\ldots}$

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Pls resend! $\longleftarrow$

Reaction attack

$\mathbf{m}_1, \mathbf{e}_1, \mathbf{c}_1 = \mathbf{m}_1\mathbf{G} + \mathbf{e}_1$ $\xrightarrow{\mathbf{c}_1}$ ✓ $\leftarrow \mathit{Decode}(\mathbf{c}_1)$

$\mathbf{m}_2, \mathbf{e}_2, \mathbf{c}_2$ $\xrightarrow{\mathbf{c}_2}$ ✓ $\leftarrow \mathit{Decode}(\mathbf{c}_2)$

$\xrightarrow{\quad\cdots\quad}$

$\mathbf{m}_t, \mathbf{e}_t, \mathbf{c}_t$ $\xrightarrow{\mathbf{c}_t}$ X $\leftarrow \mathit{Decode}(\mathbf{c}_t)$

Pls resend! $\longleftarrow$

Key recovery attack

When does a decoding failure happen?

(A closer look at) the syndrome equation for LRPC:

$$\mathbf{H}_{secret}\mathbf{e}^\top = \mathbf{s}$$

$$s_i = \sum_{j=1}^{n} h_{i,j} e_j = \sum_{j=1}^{n} \left(\sum_{l=1}^{d} h_{i,j,l} F_l \right) \left(\sum_{v=1}^{r} e_{j,u} E_u \right)$$

$$= \sum_{l=1}^{d} \sum_{u=1}^{r} F_l E_u \left(\sum_{j=1}^{n} h_{i,j,l} e_{j,u} \right), \quad \forall i \in \{1, \ldots, n-k\}.$$

In matrix form:

$$\mathbf{s} = (F_1 E_1, F_1 E_2 \ldots, F_d E_r) \cdot \bar{\mathbf{A}}_{\mathbf{h},\mathbf{e}}$$

Recall: Decoding fails when $\mathrm{Dim}(S) < rd$

$\bar{\mathbf{A}}_{\mathbf{h},\mathbf{e}}$ **is not of full rank**

Our Attack

What to do with the errors?

$$\begin{cases} \mathbf{v}_{\mathbf{e}_1} \cdot \bar{\mathbf{A}}_{\mathbf{e}_1}(\mathbf{h}) = \mathbf{0}_{1\times n-k} \\ \mathbf{v}_{\mathbf{e}_2} \cdot \bar{\mathbf{A}}_{\mathbf{e}_2}(\mathbf{h}) = \mathbf{0}_{1\times n-k} \\ \dots \\ \mathbf{v}_{\mathbf{e}_t} \cdot \bar{\mathbf{A}}_{\mathbf{e}_t}(\mathbf{h}) = \mathbf{0}_{1\times n-k} \end{cases}$$

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High level attack idea:

0: *Collect errors* $\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_t$ *from decryption failures*
0: **repeat**
0: $\quad \mathbf{h} \leftarrow \text{SolveSystem}(\mathbf{v}_{\mathbf{e}_1}, \mathbf{v}_{\mathbf{e}_2}, \dots, \mathbf{v}_{\mathbf{e}_t}, \mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_t)$
0: $\quad$ **if** $\mathbf{h} \neq \perp$ **then**
0: $\quad\quad$ *Collect* ℓ *messages, errors, ciphertexts* $(\mathbf{m}_i, \mathbf{e}_i, \mathbf{c}_i)$
0: $\quad\quad F, \text{success} \leftarrow \text{FindBasis}(\mathbf{h}, \{(\mathbf{m}_i, \mathbf{e}_i, \mathbf{c}_i)\}_{i=1}^{\ell})$
0: $\quad$ **else** $\text{success} \leftarrow \perp$
0: **until** success
0: $\mathbf{H} \leftarrow \text{ReconstructMatrix}(\mathbf{h}, F)$
return $\mathbf{H}$ of small rank d =0

Our Attack

How to solve the system?

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- Kernel method
  - $n-k$ equations for each error $\mathbf{e}_i$
  - nd unknown coefficients in $\mathbf{h}$
  - guess $\mathbf{v}_{\mathbf{e}_i}$ in kernel of $\bar{\mathbf{A}}_{\mathbf{e}_t}(\mathbf{h})$
  - $\Rightarrow$ linear system only in the nd $\mathbf{h}$-variables
  - need to collect $t \geqslant \frac{nd}{n-k}$ errors from DF

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  - Probability of guessing $\mathbf{v}_{\mathbf{e}_i}$ correctly: $P_{\mathbf{e}_i} = \dfrac{q^{K_{\mathbf{e}_i}}}{q^{rd}}$, where $q^{K_{\mathbf{e}_i}} = |Ker(\bar{\mathbf{A}}_{e_i}(h))|$.

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  - Probability of guessing all $\mathbf{v}_{\mathbf{e}_1}, \dots, \mathbf{v}_{\mathbf{e}_t}$: $P_t = P_{\mathbf{e}_i}^t = q^{-(rd-1)t}$.

Our Attack

Can it be improved?

An LRPC cryptosystem, with a secret key $\mathsf{sk} = (\mathbf{H}, \cdot)$ has an *equivalent key* $\mathsf{sk}' = (\mathbf{H}', \cdot')$, if $\mathsf{sk}' \neq \mathsf{sk}$ and sk' can be used as a secret key with equal efficiency as sk. In particular, $\mathbf{H}'$ is of the same rank as $\mathbf{H}$.

- If $\mathbf{W} \in GL_n(\mathbb{F}_q)$, $\mathsf{sk}' = (\mathbf{WH}', \cdot')$ is an equivalent key.
- Decryption failures are invariant with respect to equivalent keys
- We can rewrite $\mathbf{H}$ as

$$\mathbf{H} = \sum_{i=1}^{d} \hat{\mathbf{H}}_i \cdot F_i = \sum_{i=1}^{d} [\hat{\mathbf{H}}_{i1} | \hat{\mathbf{H}}_{i2}] \cdot F_i$$

$$\Rightarrow \mathbf{H}' = [\mathbf{I}_{n-k} | \hat{\mathbf{H}}'_{12}] \cdot F_1 + \sum_{i=2}^{d} [\hat{\mathbf{H}}'_{i1} | \hat{\mathbf{H}}'_{i2}] \cdot F_i \text{ is an equivalent key.}$$

- **We reduce the number of variables to $nd - (n-k)$.**

Our Attack

Our attack on McNie

- We evaluated the attack on the 1st round submission parameters

n	k	d	r	q	m	Dec. Failure	Security (bits)	Classical Attack (bits)	Quantum Attack (bits)	t
93	62	3	5	2	37	2^{-17}	128	138	82.8	8
105	70	3	5	2	37	2^{-20}	128	140	83.7	8
111	74	3	7	2	41	2^{-17}	192	188	108	8
123	82	3	7	2	41	2^{-20}	192	189	109	8
111	74	3	7	2	59	2^{-17}	256	188	108	8
141	94	3	9	2	47	2^{-20}	256	238	134	8

- Better attacks exist -
  We do not take advantage of any additional structure of McNie
- We do not take full advantage of the high decryption failure

Conclusion

Final notes

- Reaction attacks in the Hamming metric - Guo, Johansson, Stankovski '16;
- A concurrent work in the Rank metric - Aragon, Gaborit '19;

Differences to our attack

- We need only a handful of observed decryption failures
- We do not rely on any statistical tests
- We do not rely on any specific decoder
- We assume "randomly generated" errors

Questions

Thank you for your attention.
Questions?
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