

# How to Sign with White-Boxed AES



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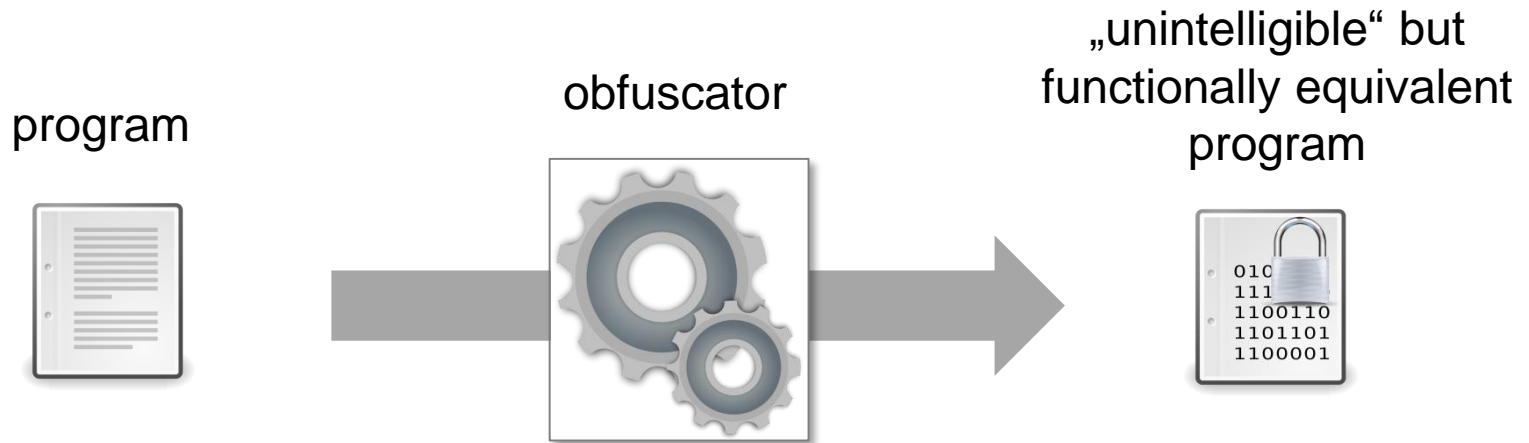
Latincrypt 2019

Marc Fischlin

joint work with  
Helene Haagh

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# Obfuscation

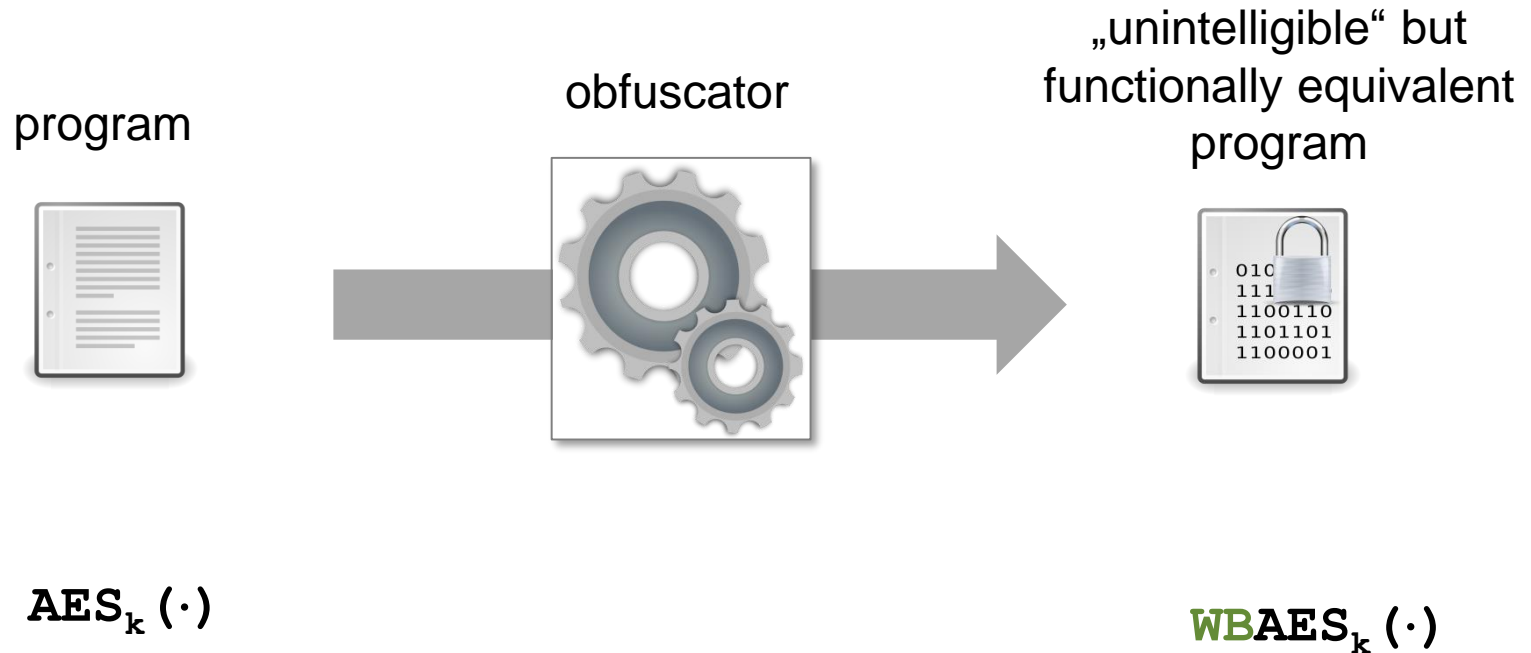


```
alert('Hello, World!')
```

```
eval(function(p,a,c,k,e,d){e=function(c){return c};if(!''.replace(/^/,String)){while(c--){d[c]=k[c]||c}k=[function(e){return d[e]};e=function(){return'\\w+'};c=1};while(c--){if(k[c]){p=p.replace(new RegExp('\\b'+e(c)+'\\b','g'),k[c])}}return p}('0(\\'1,2!\\')',3,3,'alert|Hello|World'.split('|'),0,{}))
```

# White-Boxing AES

Chow, Eisen, Johnson, van Oorschot (SAC 2002)



In particular, white-box version shall hide the secret key

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# White-Boxing is hard...

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WhibOx Contests in 2017 and 2019: Competitors only lasts days or weeks

Even unclear if theoretical solutions exist at all

**...but not the topic of this talk**

Instead: How to use a good white-boxing

# When to White-Box?

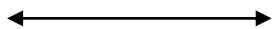


$\text{WBAES}_k(\cdot)$

$\text{AES}_k(\cdot)$

protects key  $k$   
against leakage on device

schemes based on  
shared symmetric key  $k$



$\text{AES}_k(\cdot)$

$\text{WBAES}^{-1}_k(\cdot)$

$\text{AES}_k(\cdot)$ ,  $\text{WBAES}^{-1}_k(\cdot)$   
= secret-public key pair

fast signing with key  $k$  on device  
slow(er) verification on server

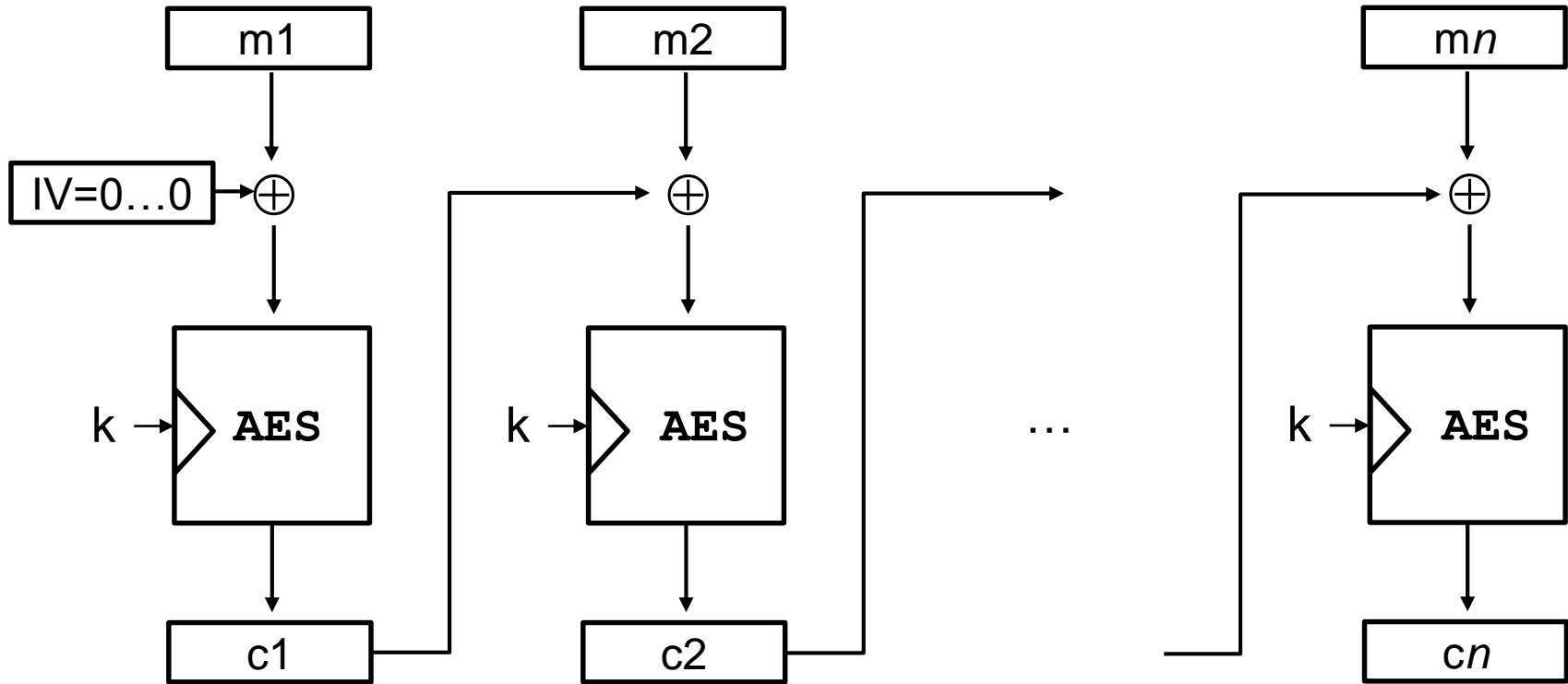
*This talk*

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# How **not** to Sign with White-boxed AES

# CBC-MAC as Signature Scheme?

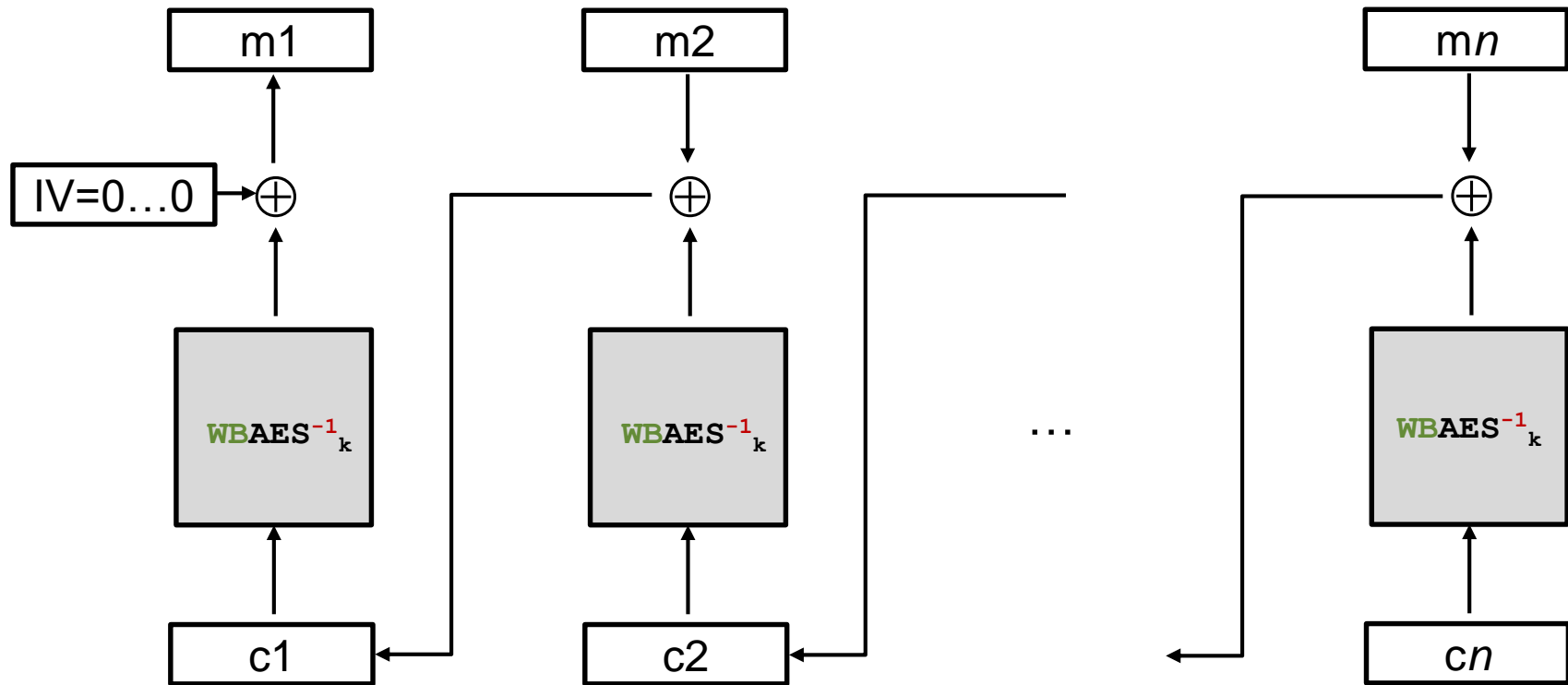
here: fixed-length,  
block-aligned messages



signature  $s=c_n$

# Verification with WBAES

given signature  $s=cn$   
and message  $m$



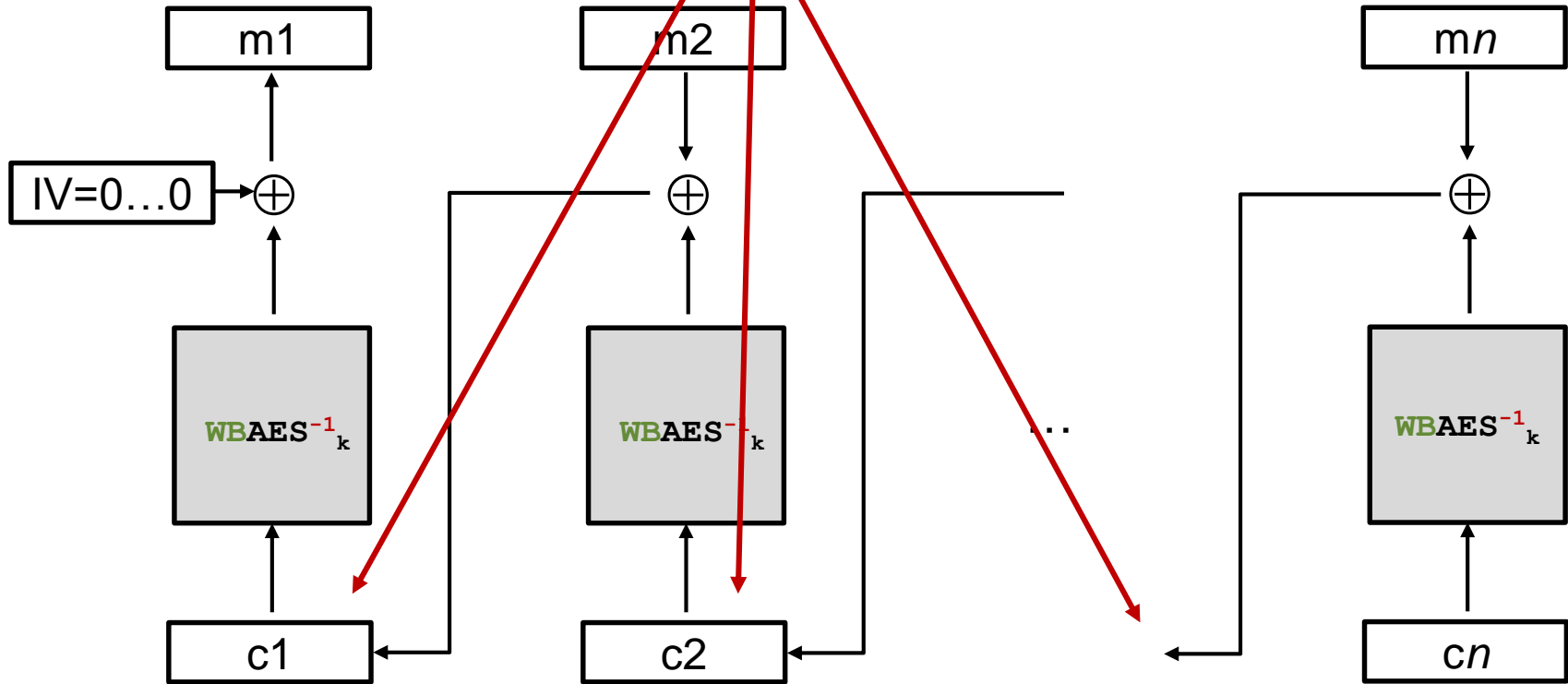
verification succeeds if „computing backwards“ yields  $m_1$



# Security?



Adversary knows public  $\text{WBAES}^{-1}_k$  and thus gets to see all intermediate results!



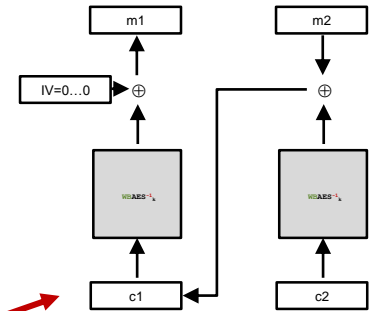
verification succeeds if „computing backwards“ yields  $m_1$

# Breaking CBC

here: with two message blocks



goal: create forgery for  $m1||m2$  with probability 1



1. get signature for  $m1||x2$ , compute intermediate result  $\text{AES}_k(m1)$
2. get signature for  $x1||0..0$ , compute intermediate result  $\text{AES}_k(x1)$
3. get signature for  $x1|| (m2 \oplus \text{AES}_k(m1) \oplus \text{AES}_k(x1))$

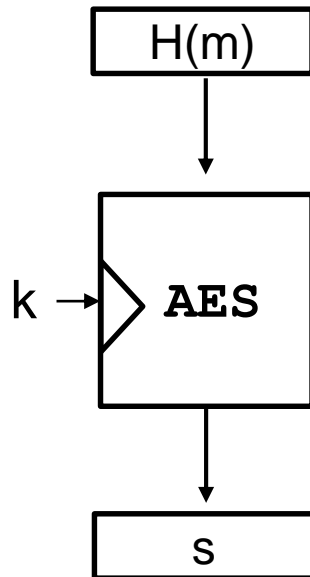
This is also a valid signature  
for the (fresh) message  $m1||m2$

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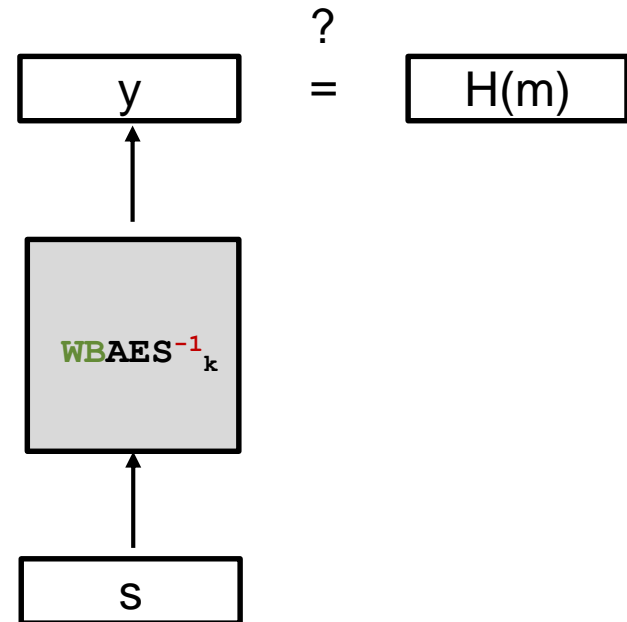
# Signing with Full-Domain-Hashing

# Full-Domain-Hash Signatures for WBAES

$\text{AES}_k(\cdot)$  = secret signing key



$\text{WBAES}^{-1}_k(\cdot)$  = public verification key



$H$  = Hash function truncated to 128 Bits

# Security Results for FDH-Signatures

Coron (Crypto 2000):

$$\text{Adv}(\text{Forge}) \leq q_s \cdot \text{Adv}(\text{Unpred})$$

$q_s = \#$ signature queries

Problem:

trivial attack strategy against unpredictability,  
wins with prob  $2^{-64}$  after  $2^{64}$  WBAES evaluations  
and no AES queries

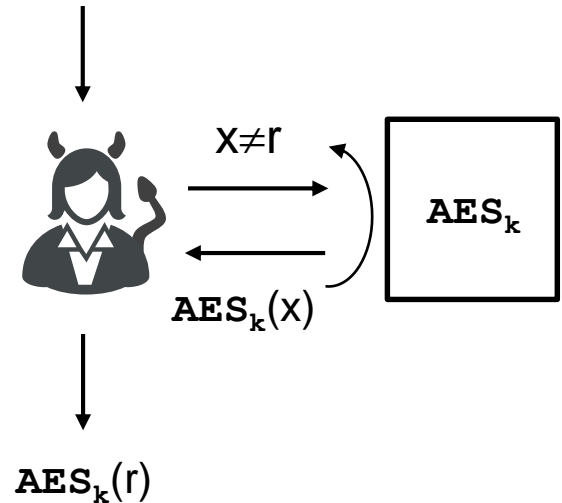
**guarantees  
for 128-bit AES?**

Other problem:

proof requires random oracle programming

Delerablee, Lepoint, Paillier, Rivain  
Unpredictability game (SAC 2013):

$\text{WBAES}_k^{-1}(\cdot)$ ,  
random  $r$



# CBC-Signatures with Random Oracles

Revisiting idea of CBC-Signing, but this time using random oracle:

Signature  $s = \text{CBC}_k(H(m))$  for  $H$  outputting multiple of 128 bits

Verification: compute CBC backwards using  $H(m)$

CBC with Random-Oracle-Hashing:

$$\text{Adv}(\text{Forge}) \leq q_H \cdot \text{Adv}(\text{Unpred}) + q_H \cdot (q_H + q_s) \cdot 2^{-128}$$

$q_H = \#$ random oracle queries

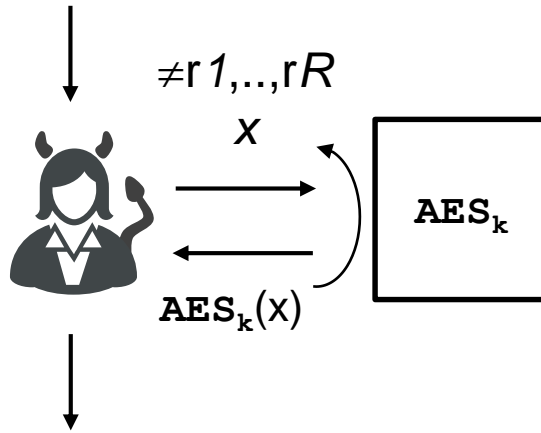
**Problem with unpredictability inherent  
due to restricted input size of AES!**

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# Correlation Intractability

# Correlation Intractability

WBAES<sup>-1</sup><sub>k</sub> (·)



correlated  $r_1, r_2, \dots, r_R$  and  
 $AES_k(r_1), AES_k(r_2), \dots, AES_k(r_R)$

according to some fixed  
(non-trivial) correlation criteria like  
 $r_1, r_2, \dots, r_R$  are equal on leading  $128 \cdot \log R$  bits

Suzuki, Tonien, Kurosawa, Toyota (ICISC'06):

generic upper bound (no WBAES input) of  
 $(q_{AES})^R \cdot 2^{-128(R-1)}$  after  $q_{AES}$  AES queries

example:  $R=4$  with  $q_{AES}=2^{64}$  yields probability of less than  $2^{-128}$



# Correlation Intractability vs. Unpredictability

Correlation intractability (for  $R=2$ )  $\Rightarrow$  Unpredictability

Correlation intractability  $\nRightarrow$  Unpredictability

for general block ciphers,  
unclear for AES

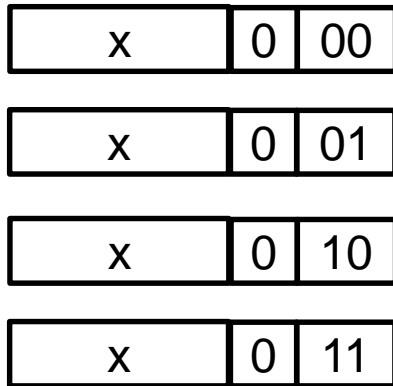
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# Signing with Chaining and Correlation Intractability

# ROChain: Signing

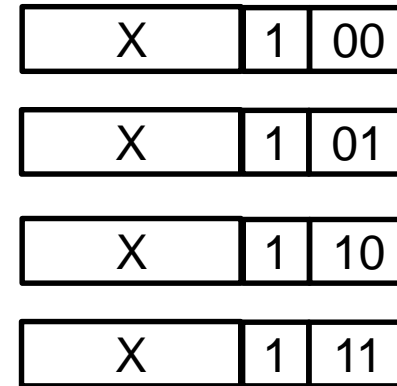
R=4

$x = H(0|m)$  truncated to 125 bits



$s = (\text{AES}_k(x|000), \dots, \text{AES}_k(x|011))$

$X = H(1|s|m)$  truncated to 125 bits



$S = (\text{AES}_k(X|100), \dots, \text{AES}_k(X|111))$

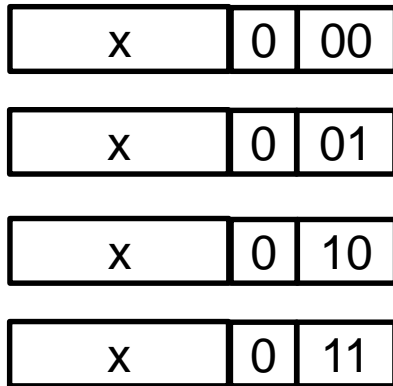
signature = ( s, S )

# Verification

recompute  $x$  and  $X$ , check each AES value, & that pre-images in  $s$  resp.  $S$  are correlated

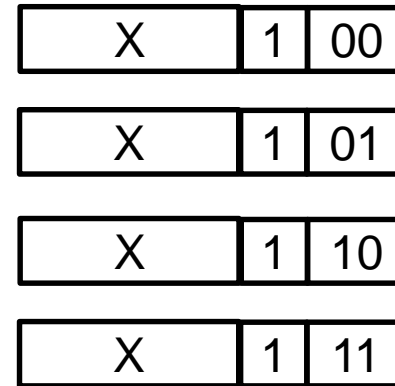
$R=4$

$x = H(0|m)$  truncated to 125 bits



$s = (\text{AES}_k(x|000), \dots, \text{AES}_k(x|011))$

$X = H(1|s|m)$  truncated to 125 bits



$S = (\text{AES}_k(X|100), \dots, \text{AES}_k(X|111))$

signature = ( s, S )

# Security (Idea)



cannot query about valid  $s$  for new  $x$  by corr.intractability

$x = H(0|m)$  truncated to 125 bits

x	0	00
x	0	01
x	0	10
x	0	11

$s = (\text{AES}_k(x|000), \dots, \text{AES}_k(x|011))$



needs to re-use  $x$  from signing query or one of very few collisions ( $\leq 2^6 \cdot q_s$ )

$X = H(1|s|m)$  truncated to 125 bits

X	1	00
X	1	01
X	1	10
X	1	11

$S = (\text{AES}_k(X|100), \dots, \text{AES}_k(X|111))$

most likely  $X^*$  for  $m^*$  different from all previous values, infeasible to find valid  $S^*$  by corr.intr.



# Security bound for ROChainSign

Security bound for ROChainSign:

$$\text{Adv}(\text{Forge}) \leq 2 \cdot \text{Adv}(\text{Corr.Intr.}) + q_s^2 \cdot 2^{-113} + 3 \cdot 2^{-128}$$

in the non-programmable random oracle model

# Extensions

Correlation intractability can be used to give counter-based and nonce-based construction without random oracles

but signatures become larger than in ROChainSig

Example ( $R=4$ , signing 256-bit messages)

Scheme	Signature Size	Random Oracle?
ROChainSig	8 AES values	non-programmable
CountSign (using counter)	16 AES values	no, but stateful
NonceSign (using nonces)	32 AES values	no, stateless

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# Conclusion



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# Conclusion (I)

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Slowing down  $\text{wBAES}^{-1}_k(\cdot)$  computations hinders attacks

similar to iterations in password-based hashing

# Conclusion (II)

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## Limited block length of AES causes trouble

bypassing problems by switching to  
correlation intractability assumption

constructions with reasonable bounds  
in non-programmable random oracle model  
and in standard model with nonces (larger signatures)

Thank you!